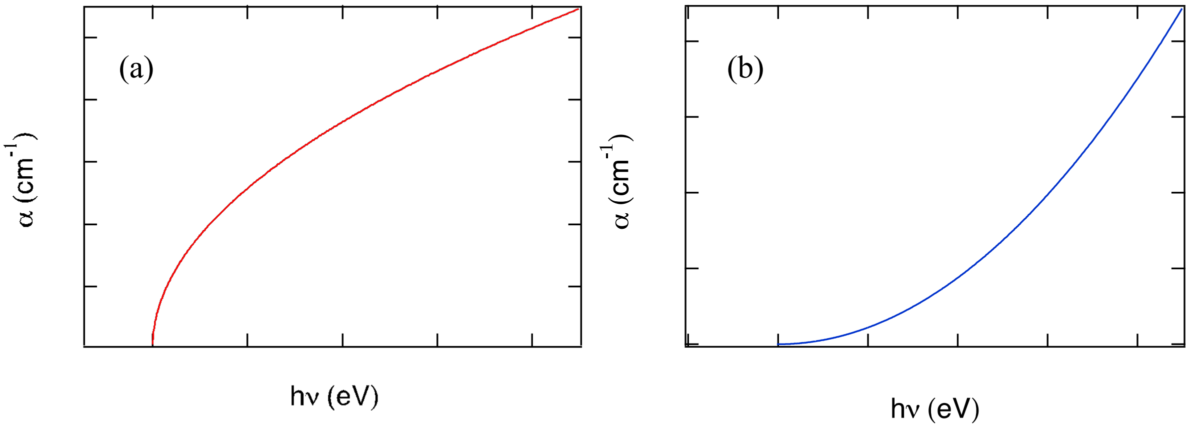
Exercice 8.1: Direct or indirect bandgap



1) The absorption coefficient has been plotted as a function of frequency for two different semiconductors. Explain what type of semiconductor is associated to each of these graphs.

2) How would these graphs evolve with temperature?

Exercise 8.1 solution.

1) As shown in the lecture, the optical absorption dependence on the frequency is different for direct and indirect bandgap semiconductors. For a direct bandgap, one expects an absorption coefficient described by:

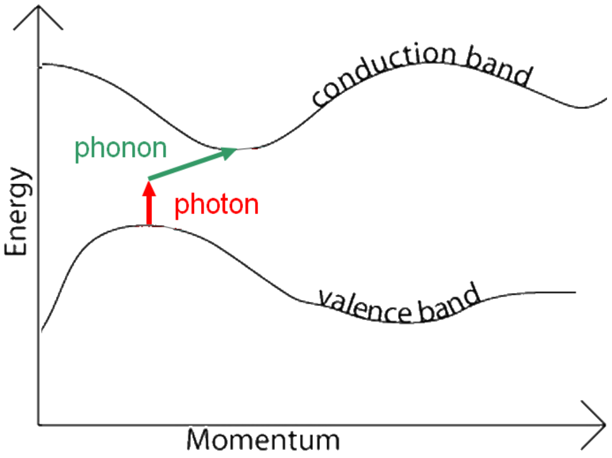
$$\alpha \sim C \cdot \sqrt{h\nu - E_g}$$

while for an indirect bandgap the expected relationship is more complicated:

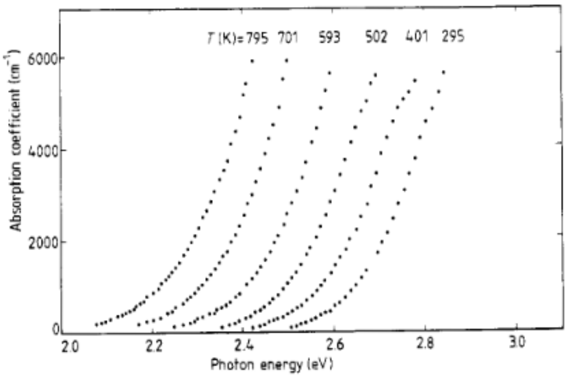
$$\alpha \sim \frac{\left(h\nu - E_g + E_p\right)^2}{\exp\frac{E_p}{kT} - 1} + \frac{(h\nu - E_g - E_p)^2}{1 - \exp\frac{E_p}{kT}}$$

By identifying the characteristic evolution in each of the graphs, one can say that graph (a) corresponds to a direct bandgap semiconductor (evolution as $y = \sqrt{x}$), while graph (b) corresponds to an indirect bandgap semiconductor (evolution as $y = x^2$).

2) As put forth in previous expressions for α, it appears that the absorption edge should not be dependent on the temperature for direct bandgap semiconductors (see note below). For indirect bandgap however, one expects a temperature dependence. This is physically meaningful: for an indirect transition, a photon and a phonon interact together to fulfil momentum and energy conservation:



The transition is energetically allowed when $E_{photon} + E_{phonon} \geq E_g$. As one increases the temperature, the phonons occupy states of higher energy (this is the Maxwell-Boltzmann weighting, that we find back in the above expressions). As a consequence of higher energy phonons, lower energy photons will be sufficient to exceed the bandgap. Therefore, the absorption edge starts at lower energy when the temperature is increased. As an example:



At (very) low temperature, only the phonon emission process is possible, which occurs for photon energies higher than the bandgap.

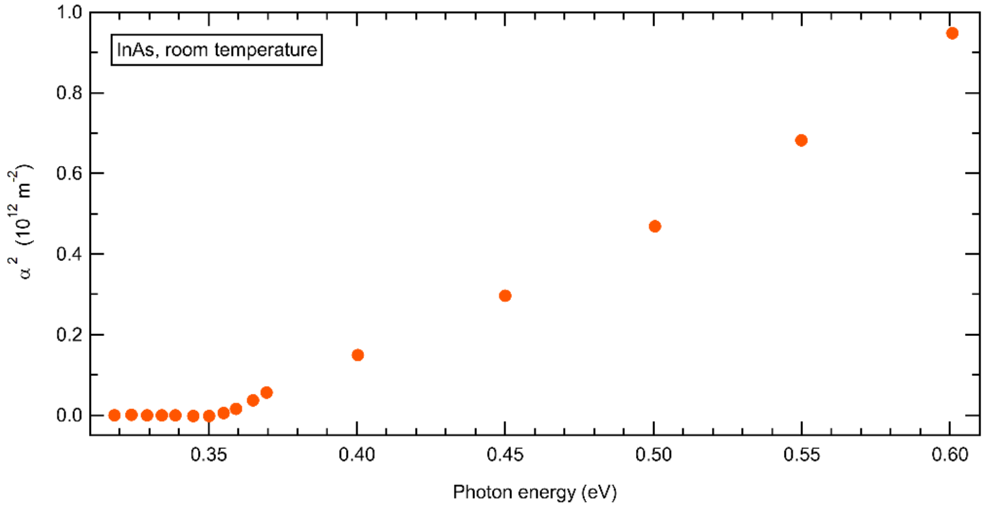
Note: In reality, besides the effect on phonon populations, an increase in temperature also results in thermal expansion of the semiconductor's lattice. This modifies the bandgap as well. For indirect bandgap this contribution is generally negligible, for direct bandgap semiconductors, this can modify the absorption edge. This behaviour is empirically described by Varshni's equation.

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta}$$

Where $E_g(0)$, α and β are material constants.

Exercice 8.2: Bandgap determination

In the figure below is plotted the absorption coefficient as a function of the photon energy for InAs. Based on this plot, is the semiconductor direct or indirect? Determine its bandgap.



Exercise 8.2 solution.

This time, α^2 is plotted, and exhibits a linear behaviour in this region close to the bandgap. Based on the expressions above, InAs is a direct bandgap semiconductor. Its absorption coefficient can be described by $\alpha \sim C \cdot \sqrt{h\nu - E_g}$ for $h\nu > E_g$, and $\alpha = 0$ for $h\nu < E_g$. Thus, since we plot α^2, the linear interpolation obtained for $h\nu > E_g$ crosses the x-axis for $h\nu = E_g$. We deduce a direct bandgap of around 0.35 eV.

Exercice 8.3: Transmission of a sample

Indium phosphide is a direct gap III–V semiconductor with a band gap of 1.35 eV at room temperature. The absorption coefficient at 775 nm is 3.5×10^6 m $^{-1}$. A platelet sample 1 µm-thick is made with anti-reflection coated surfaces. Estimate the transmission of the sample at 620 nm.

Exercise 8.3 solution.

The sample is anti-reflection coated, and so we do not need to consider multiple reflections. We therefore calculate the transmission from $T = \mathrm{e}^{-\alpha l}$ because there is no reflection. The wavelength of 775 nm corresponds to a photon energy of 1.60 eV, which is greater than E_g. Similarly, 620 nm corresponds to a photon energy of 2.00 eV, which is also above E_g. We can therefore use that $\alpha \propto \sqrt{h\nu - E_g}$, so:

$$\frac{\alpha(620\,nm)}{\alpha(775\,nm)} = \frac{\sqrt{2.00 - E_g}}{\sqrt{1.60 - E_g}} = 1.6$$

where we have used E_g = 1.35 eV. This implies that $\alpha(620\,nm) = 5.6 \times 10^6 m^{-1} = 5.6\,\mu m^{-1}$, and hence that $\alpha l = 5.6$. We thus obtain the final result:

$$T(620\,nm) = \exp(-\alpha l) = 0.37\%$$

Note: The value of T calculated in this example is only an estimate because we have ignored the excitonic effects and we have assumed that the parabolic band approximation is valid, even though we are quite a long way above E_g. The experimental value of $\alpha(620\,nm)$ is actually about 15% larger than the value calculated here.